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Innovative Applications of O.R.

Integrated location-inventory modelling under forward and reverse product flows in the used merchandise retail sector: A multi-echelon formulation

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ABSTRACT

This study presents a joint three-echelon location inventory model for a donation-demand driven industry in which the main warehouse (MW), distribution centers (DC), retail stores (RS) and donation-only centers (ADCs) exist. This unique inventory-location problem involves demand and supply uncertainties, coverage radius limitations, service level requirements, and multiple products consideration. Each retailer has two classes of products flowing from the assigned DC due to demands minus donations occurring in that retailer. The proposed model simultaneously determines the number of DCs to open, DC locations, and assignments of retailers to the open DCs for particular product types. The objective is to minimize the total annual cost including: facility location costs, transportation costs, inventory costs, and the lost sale costs. Due to the complexity of the problem, the proposed model structure allows for relaxing complicating constraints through recourse to Lagrangian relaxation. The use of robust branch-cut and price heuristics solves the mixed integer nonlinear problem to obtain a lower bound and a distance-based heuristic to get an upper bound. We formulate essential features of this novel problem, solve several numerical example problems and evaluate solution performance. We believe this is a novel problem environment, and that this initial study extends integrated location-inventory modeling to a new context.

1. Introduction

Managing product inventories by assigning correct quantities to specific locations is what links the Make, Move, Store, and Sell functions of consumer and industrial companies' supply chains. Integrating these location and inventory decisions often has significant impact on financial and operational performance due to the randomness in supply and demand (Schmit, Snyder, & Shen, 2010; Shen & Qi, 2007; Zipkin, 2000). Our work is routed in a specific problem found in the 4 trillion dollars retail sector where product supply and demand flows in a multi-echelon inventory system are stochastic.

The specific problem considered is situated in the 16 billion dollars Used Merchandise Retail (UMR) sector. This sector is comprised of both small, local operators, and national UMR chains operating as either niche retailers specializing in specific categories of items (e.g., Play-It-Again Sports, or Once Upon a Child), or as general merchandise-type retailers (e.g., Buffalo Exchange, Salvation Army). This low-profile, yet fast-growing industry has enjoyed significant yearly growth in the range of 7% in recent years (Meyer, 2016). Indeed, the National Association of Resale Thrift Shops (NARTS) reports that the last fifteen years, and the recent economic recession in particular, have witnessed significant shifts in consumers' retail purchase decisions including explosive growth in the patronage of UMR locations (NARTS, 2016). For reasons detailed later this paper, retailers in this sector are frequently subject to logistical challenges in the form of both random inbound donations (product supply arrivals) and random customer sales (demand) in their stores. Every arriving customer may or may not bring donations, and every customer may or may not make a purchase during their store visit. Goodwill Industries International (GII) oversees 165 independent retail affiliates and is the largest UMR in the world. Of these affiliates, Goodwill Industries of Southeastern Wisconsin (GISW) is the largest retail affiliate within GII and the focus of this study.

GISW operates 74 retail sites as it engages in the receipt and/or retail sale of used clothing and shoes, furniture, books and rare manuscripts, musical instruments, office furniture, phonographs

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Table 1
Stratification of RSs by net of (historical) donations minus sales.

	(Sales greatly exceed donations) CAT1	Self-sustaining favoring sales CAT2	Self-sustaining favoring donations CAT3	(Donations greatly exceed sales) CAT4
Avg. delvy. size (Donations−Sales)*	−4500 dollars	−1000 dollars	1700 dollars	6500 dollars
Avg. number of deliveries/year	150	115	100	140
Percent of total RSs	22.6%	22.6%	34.0%	20.8%
Percent of donations	15.5%	22.4%	33.9%	28.2%
Percent of total GISW sales	22.2%	25.1%	31.8%	20.9%
Percent of total deliveries	25.6%	23.1%	28.4%	22.8%

* A negative sign indicates outbound redistribution since donations exceed sales volume. Donation value conversion rates are proprietary to the company. Donations are valued at an internally-determined, base dollar rate that remains proprietary and disguised in Table 1 above.

and phonograph records, store fixtures and sports equipment. Unlike traditional retailers that either purchase their goods through contract manufacturers or manufacture product themselves, GISW procures over 80% of its product stock through donations of used goods at its various operating sites. The randomness in donation volume and donation mix creates two persistent operational challenges. First, randomness impacts store-level sales generation capacity due to stock availability mis-matches. Second, variation in store sales and the customers' donation rates at stores and ADCs leads to stock imbalances requiring frequent transshipments or repositioning between stores.

This paper considers a three-echelon location-inventory assignment problem for a UMR distribution network. The most interesting issues of this problem are captured in a robust model based upon data obtained from the company. Our remedy to sophisticated solution approaches yielded optimal results with implication for both practice and the academic community. The results are presented in this paper which we view as making two important contributions:

(i) For the academic community, the paper models the unique reverse product flows representing used merchandise received from stores and donation centers. It details the irregular pattern of inbound donations (supply inventory), the highly seasonal and volatile retail consumer demand across a network of stores and donation centers in a novel network operating structure, as well as efficient solution procedures. Other novel operating characteristics are described as contrasts to problems in other contexts and industries.
(ii) For practitioners, it provides an insightful approach for decentralizing operations while considering the tradeoffs among cost factors under alternative priority schemes. The techniques used can be implemented as a tactical response to fluctuating donations and sales in the markets facing any affiliate in the GII system.

Current research is extended to include this novel problem environment. A new formulation and optimization algorithm are proposed in order to investigate the conditions under which the product flows, coverage radius, multi-sourcing, and risk-pooling effects impact the economics of the weekly decisions. The problem objective is to minimize total costs of locating DCs, transporting working inventory between capacitated retail locations, carrying inventory safety stock at the DCs, and lost sales due to stockout. This article is organized as follows: Section 2 describes the novel UMR problem background. A review of the relevant literature on location modeling problems and stochastic inventory follows in Section 3. An integer nonlinear programming (INLP) UMR model and its relaxations appear in Section 4. Discussion of the experiment design and the solution methodology appears in Sections 5.1 and 5.2, respectively, followed by computational results of our experiments in Section 6. Future research and current limitations appear in Section 7.

2. Background on distribution at Goodwill Industries

Goodwill Industries International (GII) operates in the United States and Canada with a primary mission of offering customized training and workforce development services for, and employment (in some cases) to disadvantaged and disabled individuals. GII is a 6 billion dollars network of over 165 independently-operating and controlled retail divisions serving non-overlapping geographic regions of varying sizes. Each decoupled division comprises a network of retail stores (RS), warehouses, and attended donation centers (ADC) exclusively serving well-defined regions in its division. Profits generated in each retail division directly support GII's nonprofit mission of workforce development for persons with disabilities. GISW is the largest independent division at GII with 74 retail locations.

Market realities such as the 2007 economic recession led to significant sales growth in the UMR sector. This growth created pressure to expand the retail network for companies like GISW. Such expansion pressure creates logistical challenges, in part, because of the stochastic nature of donations and customer demand activity so stocks must be regularly redistributed between the division's network of stores. For purpose here, we assume that GISW operates from a set of known retail ADCs and RSs. The ADCs function only to receive donated product. One or more ADCs are sometimes located in the same service area as a retail store. The product types received/sold are grouped into two primary categories: hard line (HL) products (i.e., home appliances, certain electronics, books, housewares, sporting goods etc.) and soft line (SL) products (i.e., clothing, footwear, linens, towels, books, etc.). From each RS, residents donate or buy used items. An RS is described by the dollar value of donations received minus the sales generated per time period. In this sense, GISW stores fall into one of four categories based upon the net of Donations minus Sales, volume of donations, and number of replenishment deliveries (or pickups) performed for each store location. Table 1 illustrates the continuum of store categories. CAT1 stores are net 'importers' since their annual sales volume exceeds donation volume thus requiring stock replenishment to sustain store operations. CAT2 and CAT3 stores, by comparison, have a closer balance between donations and sales values but still require replenishment (CAT2) or must redistribute some excess stocks (CAT3). Finally, CAT4 stores are net 'exporters' since excess donations are relatively largest and must be redistributed. All sellable donated items have a four-week display shelf-life. If items on display in a store are not sold within four weeks of initial display, then the items must be redistributed to another store for sell, held at central warehouse as part of inventory buildup for a future store opening, or eventually salvaged at the central warehouse. On average, stores receive deliveries or have pickup of excess donations about every 2–3 days depending on daily volumes and season of the year. The highly irregular donor supply, stochastic product mix and demand, and the environment/socio-economic influences (Pickett, 2014, Washington, 2014) make the UMR industry worthy of study.

3. Literature review

The problem introduced in this study nests within the areas of facility location, stochastic inventory, and the intersection of these streams. Researchers have considered factors such as multiple echelons, transportation modes, production technology, capacity constraints, cost structure, inventory management policy, and others (Klibi, Martel, & Guitouni, 2010). This section sharpens the focus on a stream of research related to the key aspects of the UMR problem context which contains location modeling, risk pooling and coverage radius importantly. A comprehensive review of these important research topics appears in work reported elsewhere and lies beyond our purposes here.

3.1. Location modeling with risk pooling

Facility location models were introduced over seven decades ago (Baumal & Wolf, 1958; Shen, 2007) and have been the subject of a considerable body of work in the management sciences (research approach) and supply chain design (problem context). Several research reviews have appeared over the last decade indicating renewed interest in both new methods and operational contexts. These include Melo, Nickel, and Saldanha-Da-Gama (2009), Snyder (2006), and Daskin (2011). Collectively these studies comprehensively update the literature in terms of methodological approach and problem context. There are a number of classic facility location models upon which more complex, realistic models are developed. In terms of problem contexts, there is an equally vast range of application areas including, agricultural commodities, consumer goods like toys, short-life perishable products, and various delivery services (Syam & Cote, 2010), to name a few.

Location models are generally classified as capacitated or uncapacitated, single or multi-assignment, single or multi-product, static or dynamic, and with or without routing requirements. Some earlier research considered inventory management and location modelling issues separately. Recently, Shen, Coullard, and Daskin (2003), and Daskin, Coullard, and Shen (2002) present a joint location-inventory model in which location, shipment and nonlinear safety stock inventory costs are included in the same model which they frame as location modeling with risk pooling (LMRP). The idea of risk pooling (Eppen, 1979) is also utilized in other settings to minimize the system costs and increase the utilization of specialized service capacity. Mahar, Bretthauer, and Venkataramanan (2009) demonstrate how traditional retailers and online retailers can leverage their centralized, real time supply chain information by dynamically specifying which e-fulfillment location will handle each Internet sale. Their results indicate that "virtual pooling" online inventories can substantially reduce total system cost by applying the optimal static policy. Mahar, Bretthauer, and Salzarulo (2011) consider how hospital networks with multiple locations can leverage risk pooling benefits when deciding where to position specialized services, such as magnetic resonance imaging (MRI), transplants, or neonatal intensive care. They develop an optimization model to determine how many and which hospital network's locations should be set up to deliver specialized services. Constraints on service area in location models are captured with coverage radius constraints for distribution and retail node assignments. Bhattacharya and Nandy (2013) address this issue in their maximum coverage facility location study. They consider a competitive facility location scenario where, given a set of users and a set of facilities, the objective is to open a new facility in an appropriate location that maximizes the number of users served. Their model assumes that each user takes service from its nearest facility. Further extensions to these models have incorporated multi-layer facilities, multiple planning periods, and stochastic demand and cost parameters in the location modeling problem (Klose et al., 2005).

Shen and Qi (2007) develop a model in supply chain system with stochastic demand and determine the number and location of distribution centers (DCs), along with assigning retailers' demands to the DCs. They apply routing costs instead of direct shipments which is much more realistic and use Lagrangian relaxation solution approaches. Sourirajan, Ozsen, and Uzsoy (2007, 2009) develop an integrated network design model that simultaneously considers the operational aspects of lead time (based on queueing analysis) and safety stock. Their 2007 study reported the use of Lagrangian relaxation while their 2009 study used Genetic algorithms that were subsequently compared with Lagrangian results. Ozsen, Daskin, and Coullard (2009) develop a multi-sourcing, capacitated location model with risk pooling in which they consider capacity constraints based on maximum inventory accumulation. Shen (2005) studied a multiproduct extension of LMRP. Ghezavati, Jabal-Ameli, and Makui (2009) present a new model for distribution networks considering service level constraint and coverage radius. On the other hand, the selection of nodes for new facility locations can be restricted to a finite set of available locations to satisfy customer demand (ReVelle & Eiselt, 2005), or all candidate locations have the equivalent setup cost for establishing a new facility.

3.2. Inventory modeling in the location problem

The UMR problem studied here considers location and inventory policy issues jointly due to the forward and reverse product flows of donated items and the stochastic nature of not only demand but also supply. In this context the relevant literature is discussed. The inventory management research stream took the strategic facility location decisions as exogenous parameters and typically focused on minimizing the inventory costs while providing high service levels (Axsater, Marklund, & Silver, 2002; Cachon & Fisher, 2000; Marklund, 2002; Qi, Shen, & Snyder, 2009).

Parlar and Berking's (1991) study is recognized among the earliest studies to examine the stochastic nature of supply in inventory models. Using EOQ-based models, they examine the link between expected costs and order size. They assume the status of supply availability is known. Subsequent studies offer updates and extensions to the work (Ross, Rong, & Snyder, 2008). Nozick and Turnquist (2001) extend their previous work in the area by incorporating inventory costs to the location problem and assuming a Poisson-distributed demand arrival and a base stock inventory policy (lot-for-lot ordering system). They use an approximation of inventory costs in the objective function that minimizes inventory costs along with the costs of unfulfilled demands. These costs are incorporated into the fixed installation costs with additional coverage radius restrictions. Shen et al. (2003), and Daskin et al. (2002) present a joint location-inventory model in which location, shipment and nonlinear safety stock costs are considered. This research was the basis for several follow-up studies (Balcik, 2003; Shen, 2005; Shu, Teo, & Shen, 2005; Teo & Shu, 2004). Ozsen, Coullard, and Daskin (2008) propose a capacitated-single commodity LMRP. Their model considers varying costs at each DC as well as the interdependency between inventory levels and capacity constraints based on maximum inventory accumulation. Using Lagrangian relaxations, they show that DCs increase their order frequency in direct response to rising demand in the service area. By contrast though, traditional solutions of this problem involved re-allocating the additional demand to remote DCs or adding more DC locations to the system. This presents a tradeoff between risk pooling and ordering costs in the context of growing demand.

While our research parallels the relevant literature above on joint location-inventory modeling and coverage radius with stochastic demand, there are significant differences calling for additional work on generalized models and solution procedures.

Most of the work assumes that stochastic supply can be mitigated by spreading risk among several suppliers. This work also does not consider the frequent need to re-distribute (or transship) stock in the system. More generally, each retail location and donation center locations offer a novel context that has yet to appear in the literature. The UMR problem offers contributions to the literature in these respects.

We model the UMR problem as a three-echelon distribution system with specific product flows in the supply chain network (i.e., delivery shipments between RSs and DCs, and between DCs and the Main Warehouse, which are determined based on annually average of demands minus donations between retailers assigned to given DCs). Our formulation allows for reverse flows between RSs and DCs, and leads to a unique modeling approach that can be applied to other donation-driven UMR operators. The model also incorporates both multi-product and multi-sourcing extensions. The risk-pooling approach is considered for analyzing the stochastic demand and donations facing retail stores, the stochastic donations for attended donation centers, and the safety stock implications of the demand facing the DCs. We integrate the facility location decisions for opening new DCs at potential nodes with the inventory decisions impacting management of safety stock levels at the chosen DCs. With this model, it is possible to show both the systemic cost implication of introducing an upper limit on coverage radius for DC-RS, and the impact of DC-ADC assignments on the system cost. The model also considers setting limits on the minimum number of RSs and ADCs that can be assigned to each DC for the product types considered in this study. In the next section, we first formulate the problem as an integer nonlinear programming (INLP) problem, which has square root terms in the objective function (making the problem NP-hard). A Lagrangian relaxation method embedded in two-step reformulation of the original problem is then provided, which increases the efficiency of a branch-cut and price solution algorithm.

4. Model formulation

In the UMR sector, private citizens donate their reusable goods via a RS or an ADC. Reusable products received at ADCs are only transported outbound to the Main Warehouse. Only RS assignments are constrained by a coverage radius to the assigned DCs; ADCs are not to be assigned to any DC. ADCs do not generate any demand to be satisfied and neither do they fulfill RSs demands directly. This is because no transshipment is allowed among RSs or ADCs. As a result, all donations to an ADC are sent back to the Main Warehouse which has no significant holding cost to be taken account for in our model.

For the RSs, the flow is more complex in many ways (see CAT1, CAT2, CAT3 or CAT4 classification discussed earlier). First, the stochastic nature of retail demand and community donations requires policies on safety stocks for HL and SL products across multiple facilities. Second, the risk-pooling effect allows us to consider each retailer's orders independently to minimize its own expected cost. Safety stock is generally formulated as $Z_\alpha\sqrt{E(L)\sigma_D^2 + (E(D))^2\sigma_L^2}$ in which $E(L)$ and $E(D)$ are expected lead time and demand (annualized as appropriate), and σ_D^2 and σ_D^2 are the variance of lead time and demand (both annualized as appropriate), respectively. Lead times are considered known since locations within the division are well-defined. As a result, the safety stock for each DC or facility is presented as $Z_\alpha\sqrt{L\sigma_D^2}$ that σ_D^2 is the total variance of demand (annually) from all retailers assigned to given DC or facility. Thus it can be shown that the total safety stock for the system in the centralized model would be $z_\alpha\sqrt{\sum_{i=1}^{N}\sigma_i^2}$. We therefore explore how risk pooling might provide significant savings in terms of the safety stock levels in the system.

The network diagram for our problem is illustrated in Fig. 1. The overarching goal is to find cost-minimizing solutions for the system (total system costs include facility location, transportation and inventory costs). We develop an integer nonlinear programming (INLP) model, where RS assignments are constrained by a coverage radius to the assigned DCs. Relaxations of the initial model are then examined in order to enhance the efficiency of our solution algorithm. Various settings of the coverage radius, along with weight multipliers on transportation and inventory costs are captured in our experimental design that is discussed later. Sensitivity analysis examines the cost implications and components of various supply chain network decisions. For more realistic assignments we also assume that every new DC should support at least five retail locations for at least one product type. In this regard, the typical UMR supply chain network is depicted in Fig. 1.

The model assumptions, parameters and variables are defined as follows:

Assumptions:

- Inventory Policy: Continuous review with Q^* economic order quantity.
- Order lead time from main warehouse to DCs is constant.
- Minimum number of retailers/ADCs serviced by a DC for a given product: 5.
- Echelons in the supply chain network: 3 levels (Main Warehouse, intermediate DCs, and Retailers/ADCs.
- Demand and Donation flows into RSs.
- Direct shipments from the Main Warehouse to intermediate DCs and from DCs to retailers.
- Multiple products: 5 product types considered (HL: home appliances, electronics and housewares, SL: clothing, footwear).
- Includes a coverage radius constraint.
- Normally distributed demand and donation.
- RSs do not keep any safety inventory in their stores.
- We assume the RSs generally experience more demand than donation volume.

Sets/indices

I	set of RSs indexed by i
J	set of candidate DC site indexed by j
K	set of product types indexed by k

Parameters

f_j	fixed annual cost of locating a DC at candidate site j
d_{ij}	distance between DC_j and RS_i
D_{ik}	annual demand of RS_i for product k
S_{ik}	annual donation of RS_i for product k
$\mu_{D_{ik}}$	mean annual demand of RS_i for product k
$\mu_{S_{ik}}$	mean annual donation of RS_i for product k
$\sigma^2_{D_{ik}}$	variance of annual demand of RS_i for product k
$\sigma^2_{S_{ik}}$	variance of annual donation of RS_i for product k
σ^2_{ik}	variance of the difference between annual demand and donation from RS_i for product k
α	desired probability of retailer orders satisfied
Z_α	standard normal deviate such that $\Pr(z \le Z_\alpha)\alpha$
β	weight factor assigned to transportation costs
θ	weight factor assigned to inventory costs
γ	weighted factor assigned to fixed location costs
λ	weighted factor assigned to lost sale costs
$L(Z)$	standard loss function (if $\boldsymbol{Z_\alpha} = 1.96$ or $\boldsymbol{\alpha} = 97.5\%$ then $\boldsymbol{L}(\boldsymbol{Z}) = 0.0094$ from Standard Normal Loss Table)
s_k	annual lost sale unit cost for product k
CR	maximum distance allowed (coverage radius) between RS_i and assigned DC_j

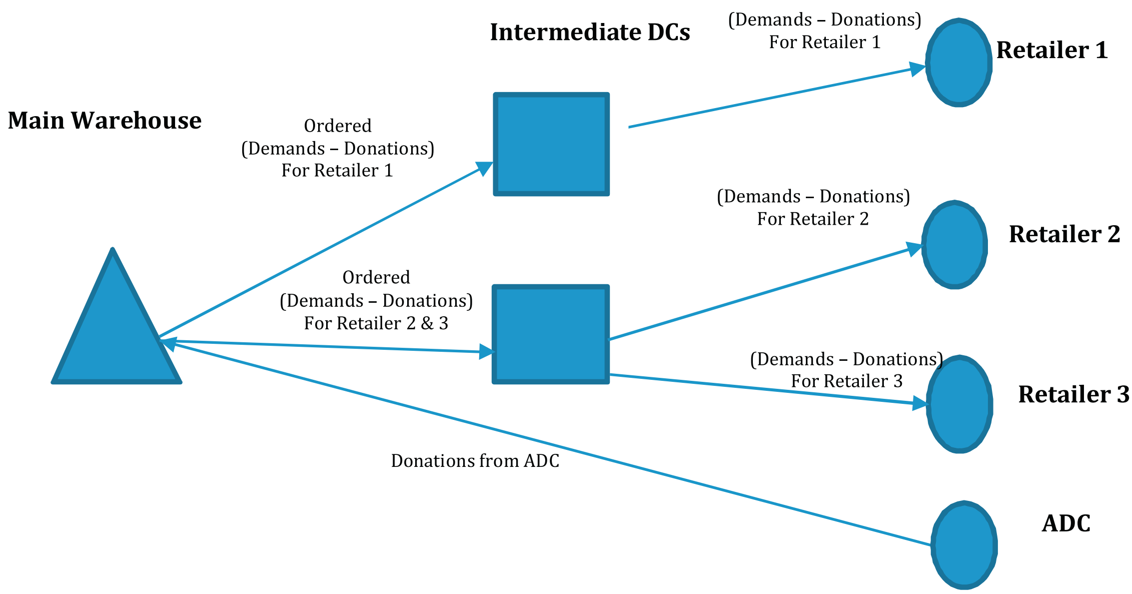

Fig. 1. Illustrative UMR supply chain network.

r_{ij}	1 if the distance between RS_i and DC_j is less than given coverage radius, 0 otherwise m = minimum number of RSs assigned to each DC for at least one product type
F_j	fixed cost of placing an order from DC_j to the main warehouse
L_j	lead time for deliveries from the main warehouse to the distribution center j (same lead time for any DC, L)
c_k	unit cost of transportation per mile per unit product (e.g., dollar cost per mile per Gaylord box)
h_k	annual holding unit cost for product k
g_j	fixed cost of transportation from main warehouse to DCs
l_j	distance from main warehouse to DCj

Decision variables

X_j	1 if we locate a DC in candidate site j, and 0 otherwise
Y_{ijk}	1 if RS_i or ADC_i is assigned to the DC_j for product type k, and 0 otherwise

4.1. UMR objective function

The objective function minimizes the total fixed cost for locating facilities, plus the weighted transportation costs from DCs to RSs, weighted transportation costs from main warehouse to DCs, working inventory cost of the supply chain network, safety stock costs at the DCs, and lost sales costs at the DCs. It is given by:

$$\begin{aligned} Z = \gamma \sum_{j\epsilon J} f_j X_j + \ \beta \sum_{j\epsilon J}\sum_{i\epsilon I}\sum_{k\epsilon K} c_k d_{ij}\left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} \\ + \beta \sum_{j\epsilon J}\sum_{i\epsilon I}\sum_{k\epsilon K} c_k l_j \left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} + \\ \sum_{j\epsilon J}\sum_{k\epsilon K}\sqrt{2\theta\left(F_j + \beta g_j\right) h_k \sum_{i\epsilon I}\left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk}}\ \frac{(p_k^2+1)}{2\ p_k} \\ + \theta Z_\alpha \sum_{j\epsilon J}\sum_{k\epsilon K} h_k \sqrt{L \sum_{i\epsilon I} \sigma_{ik}^2 Y_{ijk}} \\ + \lambda L(Z) \sum_{j\epsilon J}\sum_{k\epsilon K} s_k \sqrt{L \sum_{i\epsilon I} \sigma_{ik}^2 Y_{ijk}} \end{aligned} \tag{1}$$

If $\sqrt{\frac{h_k+s_k}{s_k}} = p_k$
where the first term accounts for the total annual fixed cost for locating facilities, the second term accounts for total annual transportation costs from DCs to RSs. The subtraction $(\mu_{D_{ik}} - \mu_{S_{ik}})$, gives the total units to be transported between the RS_i and the DC assigned for product type k. If RS_i is assigned to DC at location j, then the product of this subtraction with unit transportation cost $c_k d_{ij}$ leads to the annual total transportation cost for this RS for product type k. If RS_i is not assigned to DC at location j, the transportation cost is 0. Summing up these costs for all candidate DC locations, RSs, and all product types gives us the total transportation cost between DCs and RS. The third term is the total annual transportation costs from main warehouse to DCs.

The fourth term in the objective function accounts for total annual working inventory cost of the supply chain network when a DC orders inventory from supplier (herein Main Warehouse) using (r, Q) policy with service level constraints (Daskin et al., 2002, Shen 2007). It is the weighted (θ) total working inventory cost due to the difference between the total demand and donation from RSs. Each opened DC incurs this cost to replenish the demand at the RSs. We assume the mean annual demand always exceeds the mean annual donation otherwise the node becomes an ADC.

The fifth term accounts for total annual safety stock costs incurred by the DCs. We define σ_{ik}^2 as the variance of the difference between the retailer's demand and its donation from RS for product k. The sixth term in objective function accounts for annual lost sale costs incurred by the DCs. The function is composed of the product of expected number of lost sale (L(z)*standard deviation of demand in lead time) with annual lost sale unit cost.

It can be shown that:

$$Var\ (D-S) = \ \sigma_{ik}^2 = \ \sigma_{D_{ik}}^2 + \sigma_{S_{ik}}^2 - 2\ Cov(D,S)$$

$$Cov(D,S) = \rho(D,S).\sqrt{\sigma_{D_{ik}}^2}\sqrt{\sigma_{S_{ik}}^2}$$

from monthly demand/donation data for all retailers and products: $\rho(D, S) = 0.58$ so

$$Var\ (D-S) = \ \sigma_{ik}^2 = \ \sigma_{D_{ik}}^2 + \sigma_{S_{ik}}^2 - 1.16\ \sqrt{\sigma_{D_{ik}}^2}\sqrt{\sigma_{S_{ik}}^2} \tag{2}$$

and assuming that the demand and donation at each RS follows a normal distribution. This leads to an expression for the safety stock required in the DC at candidate location j to ensure that stock outs do not occur with a probability of α or less. Safety stock is weighted by the same factor (θ) used for the working inventory cost. The risk pooling effect implies that we take the square root of the sum of total variance in demand and donation across all

RS for all the product types to identify the gain leveraged by risk pooling (Eppen, 1979). The model constraints are discussed in the following section.

4.2. Integer nonlinear programming (INLP) problem (UMR)

The UMR Integer Nonlinear Programming (INLP) problem incorporates a nonlinear objective function (Eq. (1)) and system constraint Eqs. (4)–(8) described as follows:

(P0) – UMR (INLP) Model

$$\text{Minimize } Z \tag{3}$$

subject to

$$Y_{ijk} \le r_{ij} X_j, \;\; \forall \; i\epsilon I, \;\; j\epsilon J, \;\; k\epsilon K \tag{4}$$

$$\sum_{j\epsilon J} Y_{ijk} = 1, \;\; \forall \; i\epsilon I, \;\; k\epsilon K \tag{5}$$

$$\sum_{i\epsilon I} Y_{ijk} \ge m X_j, \;\; \forall \; j\epsilon J, \;\; k\epsilon K \tag{6}$$

$$X_j \in \{0,1\}, \;\; \forall \; j\epsilon J \tag{7}$$

$$Y_{ijk} \in \{0,1\}, \;\; \forall \; i\epsilon I, \;\; j\epsilon J, \;\; k\epsilon K \tag{8}$$

- The first constraint (Eq. (4)) requires that each RS_i has to be within the coverage radius of an opened DC_j that is assigned to it. Our model assigns values for r_{ij} depending on the following logic:

  $$r_{ij} = \begin{cases} 1, & d_{ij} \le CR \\ 0, & d_{ij} > CR \end{cases}$$

- The second constraint set (Eq. (5)) ensures that the flow of each product type in a RS can only be assigned to a single DC. Note that, we allow different product types in a RS to be assigned to different DCs in the supply chain network.
- The third constraint set (Eq. (6)) introduces a lower bound for minimum number of assignments to each DC. We define parameter m to represent minimum number of RSs assigned to each DC for at least one product type.
- Finally, constraint Eqs. (7) and (8) ensure the binary integrality of the decision variables.

The integrated distribution design and inventory management model (P0) is an INLP model with all binary decision variables. (P0) is a very difficult model to solve especially for large networks due to the potentially large number of binary variables. Applying results from Proposition 1 below reveals that the assignment variables (Y_{ijk}) in the model can be relaxed to be continuous without changing the optimal integer solution (Falk & Hoffman, 1976; You & Grossman, 2008). Therefore Model (P0) is reformulated as a mixed integer nonlinear programming (MINLP) UMR Problem (P1).

Proposition 1. *"The continuous variables Y_{ijk} yield 0–1 integer values when (P0) is globally optimized or locally optimized for fixed 0 to 1 value for each X_j."*

Proposition 1 implies that model (P1) presented below, yields integer values on the assignment variables (Y_{ijk}) when it is globally optimized or locally optimized for fixed 0–1 integer values for X_j. The proof is given in Appendix A. For a fixed set of opened DCs, problem (P1) is a concave minimization problem defined over a polyhedron. As a result, it can be shown that local and global solutions for fixed integer values of X_j yield integer values for the continuous variables.

4.3. Mixed integer nonlinear programming (MINLP) problem (P1)

Proposition 1 allows us to solve the MINLP model (P1) instead of the INLP model (P0), significantly reducing the computational effort. Replacing constraint Eq. (8) in model (P0) with constraint eq. (9) presented below gives us model (P1).

(P1) – UMR (MINLP) Model

Minimize Z

subject to (4)–(7) and:

$$Y_{ijk} \ge 0, \;\; \forall \; i\epsilon I, \;\; j\epsilon J, \;\; k\epsilon K \tag{9}$$

To further improve the computational efficiency of solving model (P1), a second reformulation of the problem is presented where the square root terms in the objective function of (P1) are eliminated by introducing two nonnegative continuous variables U_{jk} and V_{jk} such that:

$$U_{jk}^2 = \sum_{i\epsilon I} \sigma_{ik}^2 Y_{ijk}, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{10}$$

$$V_{jk}^2 = \sum_{i\epsilon I} \left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk}, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{11}$$

$$V_{jk} \ge 0, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{12}$$

$$U_{jk} \ge 0, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{13}$$

Since the nonnegative variables U_j and V_j are introduced into the objective function with positive coefficients and the problem we study is a minimization problem, Eqs. (10) and (11) can be relaxed as follows:

$$\sum_{i\epsilon I} \sigma_{ik}^2 Y_{ijk} - U_{jk}^2 \le 0, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{14}$$

$$\sum_{i\epsilon I} \left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} - V_{jk}^2 \le 0, \;\; \forall \; j\epsilon J, \;\; \forall \; k\epsilon K \tag{15}$$

4.4. Modified mixed integer nonlinear programming (MINLP) problem (P2)

Inserting the nonnegative variables U_j and V_j into the objective function and adding the non-negativity constraints 14 and 15 for U_j and V_j results in the MINLP model (P2).

(P2) – UMR (MINLP) Model

$$\begin{aligned} Z = \gamma \sum_{j\epsilon J} f_j X_j + \; \beta \sum_{j\epsilon J} \sum_{i\epsilon I} \sum_{k\epsilon K} c_k d_{ij} \left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} \\ + \beta \sum_{j\epsilon J} \sum_{i\epsilon I} \sum_{k\epsilon K} c_k l_j \left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} \\ + \sum_{j\epsilon J} \sum_{k\epsilon K} \sqrt{2\theta\left(F_j + \beta g_j\right) h_k} \; \frac{(p_k^2 + 1)}{2 \; p_k} V_{jk} + \theta Z_\alpha \sum_{j\epsilon J} \sum_{k\epsilon K} h_k \sqrt{L} \; U_{jk} \\ + \lambda L(Z) \sum_{j\epsilon J} \sum_{k\epsilon K} s_k \sqrt{L} \; U_{jk} \end{aligned} \tag{16}$$

subject to (4)–(7), (9) and (12)–(15):

(P2) can be shown to be equivalent to (P1) but now with a linear objective function and quadratic term in the constraints. The following proposition holds:

Proposition 2. *"The global optimal solution of problem (P2) or a local optimal solution with fixed 0 to 1 value for X_j, has all the continuous variables Y_{ijk} take on integer value (0 or 1)."*

Proposition 2 suggests that modified model (P2) will provide 0–1 integer values for assignment variables Y_{ijk}, when we globally optimize (P2) or achieve a local optimum with fixed 0–1 integer value for X_j. Proof of Proposition 2 is also presented in Appendix

A, which was adapted from You and Grossman (2008). As a result, we can conclude that global or local optimal solutions for model (P2) will also provide global or local optimal solutions for our original problem (P0). To have better efficiency of computational results, we will use a Lagrangian relaxation method for (P2). LR procedure adapted from Fisher (2004) has been widely applied to a vast range of location-inventory problems.

4.5. Lagrangian relaxation to P2

We relax the most sophisticated constraint (One-sourcing constraint) and add it to the objective function by Lagrangian multipliers (as penalty cost). Lagrangian relaxation method is an iteration-based algorithm which uses lower bounds (created by solving the lagrangian dual problem solution and infeasible) and upper bounds (created by lower bounds but feasible) to get optimal or near optimal solution (in Minimization problems). This algorithm stops when one of the stop criteria such as # of iterations, % gap between upper bound and lower bound, or step size is satisfied (Fisher, 2004).

(P3) – Lagrangian Dual

$$Max_\pi \; Min\; Z = \gamma \sum_{j\epsilon J} f_j X_j + \beta \sum_{j\epsilon J}\sum_{i\epsilon I}\sum_{k\epsilon K} c_k d_{ij}\left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} + \beta \sum_{j\epsilon J}\sum_{i\epsilon I}\sum_{k\epsilon K} c_k l_j\left(\mu_{D_{ik}} - \mu_{S_{ik}}\right) Y_{ijk} + \sum_{j\epsilon J}\sum_{k\epsilon K}\sqrt{2\theta\left(F_j + \beta g_j\right)h_k\, \frac{(p_k^2+1)}{2\; p_k}} V_{jk} + \theta Z_\alpha \sum_{j\epsilon J}\sum_{k\epsilon K} h_k\sqrt{L}\, U_{jk} + \lambda L(Z)\sum_{j\epsilon J}\sum_{k\epsilon K} h_k\sqrt{L}\, U_{jk} + \sum_{i\epsilon I}\sum_{k\epsilon K}\pi_{ik}\left(1 - \sum_{j\epsilon J} Y_{ijk}\right) \quad (17)$$

subject to 4, 6, 7, 9, 12–15

$$\pi_{ik} \geq 0, \;\forall\; i\epsilon I,\; k\epsilon K \quad (18)$$

For simplicity we can reformulate the model as below:

$$Max_\pi Min \sum_{j\epsilon J}\left(\hat{f}_j X_j^{*} + \sum_{i\epsilon I}\sum_{k\epsilon K}\hat{d}_{ijk} Y_{ijk} + \sum_{k\epsilon K}\hat{I}_{ijk} V_{jk} + \sum_{k\epsilon K} q U_{jk}\right) + \sum_{i\epsilon I}\sum_{k\epsilon K}\pi_{ik}\left(1 - \sum_{j\epsilon J} Y_{ijk}\right) (A12)$$

subject to 4, 6, 7, 9, 12–15 Where:

$$\hat{f}_j = \gamma f_j$$

$$\hat{d}_{ijk} = \beta c_k \;(d_{ij} + l_j)\left(\mu_{D_{ik}} - \mu_{S_{ik}}\right)$$

$$\hat{I}_{jk} = \sqrt{2\theta\left(F_j + \beta g_j\right)h_k \frac{(p_k^2+1)}{2\; p_k}}$$

$$q = (\theta Z_\alpha h_k + \lambda L(Z) s_k)\sqrt{L}$$

5. Research design and solution approach

5.1. Experimental design

Experiments are constructed for 30 and 45-node datasets randomly chosen from the existing GISW locations. The 45-node problem contains 39 and 4 RS and ADC nodes, respectively. The 30-node problem contains 26 and 2 RS and ADC nodes, respectively. In both sets, two additional candidate nodes (for each scenario) are included for locating additional DCs. As for the experimental factors, we utilized four weight parameters β, θ, γ, and $\boldsymbol{\lambda}$ for transportation, inventory, facility location costs, and lost sale costs, respectively. For each experiment, these weights are used as normalized values (summation to unity) to evaluate the effect of a change in each cost component on the total objective function and network configuration. Hence, we demonstrate realistic interactions among facility location, transportation, and inventory costs. We also use three levels for coverage radius (CR) in miles (i.e., 50, 75, 100), a range that spans the actual coverage options used in practice. The CR levels paired with the twelve weight combination sets (Table 2), lead to a mixed-level 3 by 12 factorial design with 36 experiments. Finally, we utilize a service confidence level (Z_α), {1.96}, representing $\alpha = 0.975$ as an alternative service level for the normally distributed stochastic demand in the RSs.

Table 2
Experimental design factors (model parameters).

Set#	β	θ	γ	Λ	Set#	β	θ	γ	Λ
#1	0.1	0.1	0.7	0.1	**#7**	0.2	0.4	0.2	0.2
#2	0.1	0.7	0.1	0.1	**#8**	0.2	0.2	0.2	0.4
#3	0.7	0.1	0.1	0.1	**#9**	0.4	0.3	0.2	0.1
#4	0.1	0.1	0.1	0.7	**#10**	0.1	0.2	0.3	0.4
#5	0.2	0.2	0.4	0.2	**#11**	0.2	0.4	0.1	0.3
#6	0.4	0.2	0.2	0.2	**#12**	0.3	0.1	0.4	0.2

Table 3
Product types, lines and their respective transportation, holding cost, and lost sale cost.

Product type	Product line	h_k	c_k	s_k
Home appliances	HL	0.45	1	0.1
Electronics	HL	0.8	0.78	0.2
Housewares	HL	1	1.3	0.25
Clothing	SL	1.45	0.55	0.4
Footwear	SL	1.95	1.25	0.5

As a starting point, we characterized certain model features based upon store-level operational data for ADCs and retail stores. It was determined that approximately 80% of transportation movements were between retail stores and DCs, while 20% were between all other system entities during the off-peak (non-summer months). For peak summer months, this mix fell to 60/40. The specific product line mix ratio (HL vs. SL) ranged from 66/34 for non-summer months to 35/65 for summer months. Mean demand and surplus levels for products were generated using these findings. The result is that 43.1% of the RS outbound trips move HL products, 26.9% for SL products, 15.15% are surplus HL products and 14.85% are for surplus SL products. Also 60% of trips to ADCs accounted for surplus HL products and 40% accounted for surplus SL products. The average demand/surplus at the product-level (three of HL and two of SL) is proportional to the HL/SL ratio for consistency. However, we use different unit holding and transportation costs for each product type as given in Table 3. Utilizing these annualized ratios and other relevant data, we completed the dataset construction by generating demand and donation data for each RS for different products (i.e., mean demand, $\mu_{D_{ik}}$, and donation, $\mu_{S_{ik}}$).

Next, we assumed that the coefficient of variation for mean demand and donation of HL and SL products to be a uniformly distributed random variable CV~U(0.1, 0.3) based on operational data. This allows the mean demand and donation for HL and SL products to assume a coefficient of variation between 10% and 30% around the calculated mean demand and donation values. Then, we multiplied the coefficient of variation values with the mean demand and donation values to get an estimate for the standard deviation of data (*i.e.*, $\sigma_{D_{ik}} = CV.\mu_{D_{ik}}$, $\sigma_{S_{ik}} = CV.\mu_{S_{ik}}$). Taking the square of the standard deviation values provide the variance of demand and donation for HL and SL products. Finally, Eq. (4), which is re-written below for convenience gives us the variance of the difference of

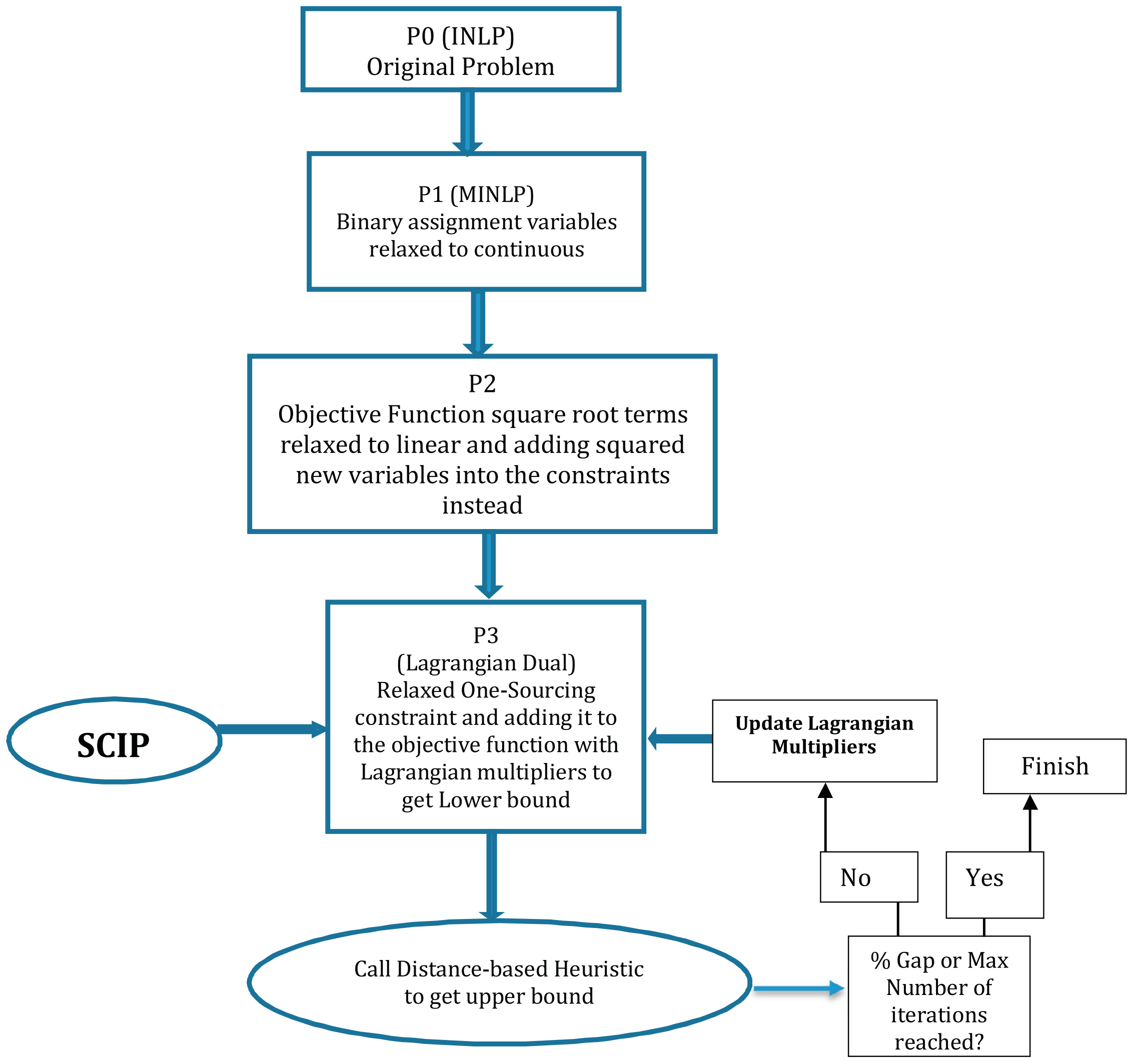

Fig. 2. Solution algorithm flowchart.

annual demand and donation from RS_i for product k, which is used to calculate the safety stock cost for the supply chain network. Total sum of variance for all product types is the same as the total sum of variance for HL and SL.

$$\sigma_{ik}^2 = \sigma_{D_{ik}}^2 + \sigma_{S_{ik}}^2 - 1.16\sqrt{\sigma_{D_{ik}}^2}\sqrt{\sigma_{S_{ik}}^2} \quad (4)$$

In order to calculate the distance between two nodes,d_{ij}, we first determined the altitude and longitude values for each location and used the *great-circle distance* formula. The great-circle distance is the shortest distance between two points on the surface of a sphere, measured along the surface of the sphere, as opposed to a straight line through the sphere's interior. Deza and Deza (2006) present empirical support for the great-circle approach to distance calculations. In the USA, the multiplier is about 1.15 in the east-west direction, and about 1.21 in the north-south direction. In this study, we used a multiplier of 1.21, since the nodes in our experiments generally oriented in north-south direction:

Other parameter values used in the experiments are as follows:

f_j=(80, 120) (i.e., representing a fixed annual cost of minimum 80,000 dollars up to maximum of 120,000 dollars for locating a DC at candidate site j).

$m = 5$ (i.e., minimum five RSs or ADCs assigned to each DC for at least one product type)

$F_j = 10$; $L_j = 1$; $g_j = 10$.

5.2. Solution methodology

5.2.1. Lower bound

In order to solve our problem (P2), we applied the SCIP (Solving Constraint Integer Programs) solver accorded to Achterberg (2009) embedded in Lagrangian Relaxation algorithm (outlined in Fig. 2). Solving P3 problem (Dual Lagrangian), gives a lower bound to the original problem, P2, in each iteration. SCIP is a mixed integer-programming solver framework that integrates constraint programming (CP), mixed integer programming (MIP), and satisfiability (SAT) techniques. The algorithm was implemented on a Dell Optiplex computer with a 2.33 gigahertz Intel i5 Dual Core processor and 4 gigabytes memory.

SCIP is based on the generally accepted *branch-and-bound procedure*. The idea of *branching* is to successively divide the given problem instance into smaller subproblems until the individual subproblems are easy to solve. The best of all solutions found in the subproblems yields the global optimum. During the course of the algorithm, a *branching tree* is created with each node representing one of the subproblems. The intention of *bounding* is to avoid a complete enumeration of all potential solutions of the initial problem, which are usually exponentially large in number. If a subproblem's lower (dual) bound is greater than or equal to the global upper (primal) bound, the subproblem can be pruned. Lower

Table 4
Results of experiments for the 30-node scenario.

Exp #	CR	Β	Θ	Γ	λ	% Gap	Time (s)	Total Obj. Value ($Thousands)	DCs Opened	# Open DCs	Lagrangian Iterations
1	100	0.1	0.1	0.7	0.1	0.00%	2	1374	{23}	1	14
2	100	0.1	0.7	0.1	0.1	0.00%	22	1277	{23}	1	32
3	100	0.7	0.1	0.1	0.1	0.00%	20	1976	{15, 19, 27, 35}	4	30
4	100	0.1	0.1	0.1	0.7	0.00%	22	1235	{23}	1	32
5	100	0.2	0.2	0.4	0.2	0.00%	32	1593	{27, 53}	2	45
6	100	0.4	0.2	0.2	0.2	0.00%	28	1884	{15, 27, 35}	3	42
7	100	0.2	0.4	0.2	0.2	0.00%	24	1566	{27, 53}	2	32
8	100	0.2	0.2	0.2	0.4	0.00%	24	1547	{27, 53}	2	34
9	100	0.4	0.3	0.2	0.1	0.00%	36	1904	{15, 27, 35}	3	48
10	100	0.1	0.2	0.3	0.4	0.00%	31	1300	{23}	1	42
11	100	0.2	0.4	0.1	0.3	0.00%	32	1543	{27, 53}	2	39
12	100	0.3	0.1	0.4	0.2	0.00%	31	1841	{27, 53}	2	39
13	75	0.1	0.1	0.7	0.1	0.00%	3	1741	{27, 53}	2	16
14	75	0.1	0.7	0.1	0.1	0.00%	25	1551	{27, 53}	2	36
15	75	0.7	0.1	0.1	0.1	0.00%	22	1976	{15, 19, 27, 35}	4	33
16	75	0.1	0.1	0.1	0.7	0.00%	27	1491	{27, 53}	2	36
17	75	0.2	0.2	0.4	0.2	0.00%	35	1593	{27, 53}	2	51
18	75	0.4	0.2	0.2	0.2	0.00%	30	1884	{15, 27, 35}	3	44
19	75	0.2	0.4	0.2	0.2	0.00%	26	1566	{27, 53}	2	33
20	75	0.2	0.2	0.2	0.4	0.00%	28	1547	{27, 53}	2	35
21	75	0.4	0.3	0.2	0.1	0.00%	43	1904	{15, 27, 35}	3	52
22	75	0.1	0.2	0.3	0.4	0.00%	35	1603	{27, 53}	2	46
23	75	0.2	0.4	0.1	0.3	0.00%	34	1543	{27, 53}	2	39
24	75	0.3	0.1	0.4	0.2	0.00%	32	1841	{27, 53}	2	39
25	50	0.1	0.1	0.7	0.1	0.00%	26	2224	{19, 27, 45}	3	19
26	50	0.1	0.7	0.1	0.1	0.00%	257	1976	{23, 37, 55}	3	38
27	50	0.7	0.1	0.1	0.1	0.00%	222	2094	{15, 27, 35, 55}	4	35
28	50	0.1	0.1	0.1	0.7	0.00%	274	1785	{23, 37, 55}	3	39
29	50	0.2	0.2	0.4	0.2	0.00%	312	2102	{19, 27, 45}	3	54
30	50	0.4	0.2	0.2	0.2	0.00%	198	2476	{9, 19, 27, 45}	4	49
31	50	0.2	0.4	0.2	0.2	0.00%	212	1862	{23, 37, 55}	3	36
32	50	0.2	0.2	0.2	0.4	0.00%	224	1845	{23, 37, 55}	3	39
33	50	0.4	0.3	0.2	0.1	0.00%	398	2624	{9, 19, 27, 45}	4	56
34	50	0.1	0.2	0.3	0.4	0.00%	322	2072	{19, 41, 45}	3	47
35	50	0.2	0.4	0.1	0.3	0.00%	260	1802	{23, 37, 55}	3	39
36	50	0.3	0.1	0.4	0.2	0.00%	188	2355	{3, 13, 19, 45}	4	42

bounds are calculated with the help of relaxation, which should be easy to solve. Upper bounds are found if the solution of the relaxation is also feasible for the corresponding subproblem. Good lower and upper bounds must be available for the bounding procedure to be effective. In order to improve a subproblem's lower bound, one can tighten its relaxation, e.g., via *domain propagation* or by adding *cutting planes*.

When the linear relaxation in each node of a branch-and-bound tree is solved by column generation, one speaks of *branch-and-price*. Optionally, as in standard branch-and-bound, cutting planes can be added in order to strengthen the relaxation, and this is called *branch-price-and-cut* (Desrosiers & Lübbecke, 2011). *Primal heuristics* have a significant relevance as supplementary procedures inside a MIP solver: they help to find good feasible solutions early in the search process, which helps to prune the search tree by bounding and allows for the application of more reduced cost fixing and other dual reductions that can tighten the problem formulation. Overall, there are 23 heuristics integrated into SCIP (Achterberg, 2009).

Generally, in each iteration of Lagrangian procedure, solving P3 problem by SCIP gives us a lower bound of our original problem, P2 (or P1, P0), and then based on a heuristic, herein Distance-based Heuristic, we obtain an upper bound for problem P2 which is feasible. This procedure continues until we reach the maximum (1000) iterations at a node, or the LB/UB gap is 0.001%. Initial Lagrange parameter settings were borrowed from Fisher (2004).

6. Computational results and discussion

We analyzed the economic benefits and the relationship between the changing values of θ, β and λ priority schemes (weights) on the desired system cost, the optimal number of open DCs, and the resulting product flow assignments (system design). A summary of the results of the 30 and 45-node problems are highlighted in Tables 4 and 5, respectively. These tables provide the overall value of the objective function (thousand dollars) as well as the number and specific locations of the DCs to open for each problem scenario. As shown earlier in Eqs. (17) and (18), parameter θ affects the contributions of the working inventory as well as safety stock due to risk pooling, β affects the contribution of the transportation cost structure and λ affects the contribution of the lost sales costs. Also reported in the results is the *% Gap which* refers to the percentage gap between the upper bound and lower bounds of the solution. 0% gap indicates that a given experiment/problem results in a global optimum. *DCs opened* indicate the nodes in which DCs are located as a part of the solution.

6.1. The number of open distribution centers

The algorithm obtained globally optimal solutions for all the 36 scenarios of the 30-node and the 45-node problems. Considering the number of open DCs, Fig. 3 provides a quick glimpse of the effect of the model parameters and variable settings. Overall, the

Table 5
Results of experiments for the 45-node scenario.

Exp #	CR	β	θ	γ	λ	% Gap	Time (s)	Obj. Value ($Thousands)	DCs Opened	# Open DCs	Lagrangian Iterations
1	100	0.1	0.1	0.7	0.1	0.00%	17	1724	{23}	1	21
2	100	0.1	0.7	0.1	0.1	0.00%	258	1634	{23}	1	32
3	100	0.7	0.1	0.1	0.1	0.00%	112	2854	{13, 16, 29, 35, 45}	5	26
4	100	0.1	0.1	0.1	0.7	0.00%	188	1587	{23}	1	31
5	100	0.2	0.2	0.4	0.2	0.00%	387	1943	{27, 53}	2	40
6	100	0.4	0.2	0.2	0.2	0.00%	276	2312	{15, 27, 35}	3	42
7	100	0.2	0.4	0.2	0.2	0.00%	223	1775	{27, 53}	2	38
8	100	0.2	0.2	0.2	0.4	0.00%	212	1743	{27, 53}	2	37
9	100	0.4	0.3	0.2	0.1	0.00%	421	2401	{15, 27, 35}	3	45
10	100	0.1	0.2	0.3	0.4	0.00%	312	1505	{23}	1	39
11	100	0.2	0.4	0.1	0.3	0.00%	356	1722	{27, 53}	2	37
12	100	0.3	0.1	0.4	0.2	0.00%	314	2172	{27, 53}	2	37
13	75	0.1	0.1	0.7	0.1	0.00%	28	1967	{12, 53}	2	25
14	75	0.1	0.7	0.1	0.1	0.00%	145	1887	{27, 53}	2	30
15	75	0.7	0.1	0.1	0.1	0.00%	262	2854	{13, 16, 29, 35, 45}	5	31
16	75	0.1	0.1	0.1	0.7	0.00%	312	1882	{27, 53}	2	37
17	75	0.2	0.2	0.4	0.2	0.00%	412	1943	{27, 53}	2	44
18	75	0.4	0.2	0.2	0.2	0.00%	332	2554	{27, 35, 53}	3	47
19	75	0.2	0.4	0.2	0.2	0.00%	267	1775	{27, 53}	2	38
20	75	0.2	0.2	0.2	0.4	0.00%	255	1743	{27, 53}	2	39
21	75	0.4	0.3	0.2	0.1	0.00%	512	2774	{27, 35, 53}	3	52
22	75	0.1	0.2	0.3	0.4	0.00%	362	1725	{27, 53}	2	42
23	75	0.2	0.4	0.1	0.3	0.00%	412	1722	{27, 53}	2	38
24	75	0.3	0.1	0.4	0.2	0.00%	356	2172	{27, 53}	2	37
25	50	0.1	0.1	0.7	0.1	0.00%	58	2645	{11, 12, 16}	3	22
26	50	0.1	0.7	0.1	0.1	0.00%	201	2106	{11, 16, 27}	3	30
27	50	0.7	0.1	0.1	0.1	0.00%	320	3243	{3, 13, 16, 29, 43, 48}	6	35
28	50	0.1	0.1	0.1	0.7	0.00%	367	2252	{11, 16, 27}	3	37
29	50	0.2	0.2	0.4	0.2	0.00%	443	2323	{11, 16, 27}	3	45
30	50	0.4	0.2	0.2	0.2	0.00%	394	2878	{16, 29, 43, 48}	4	49
31	50	0.2	0.4	0.2	0.2	0.00%	320	1996	{11, 16, 27}	3	39
32	50	0.2	0.2	0.2	0.4	0.00%	288	1925	{11, 16, 27}	3	38
33	50	0.4	0.3	0.2	0.1	0.00%	577	2956	{16, 29, 43, 48}	4	55
34	50	0.1	0.2	0.3	0.4	0.00%	388	2011	{11, 16, 27}	3	41
35	50	0.2	0.4	0.1	0.3	0.00%	454	2003	{11, 16, 27}	3	39
36	50	0.3	0.1	0.4	0.2	0.00%	402	2355	{11, 16, 27}	3	40

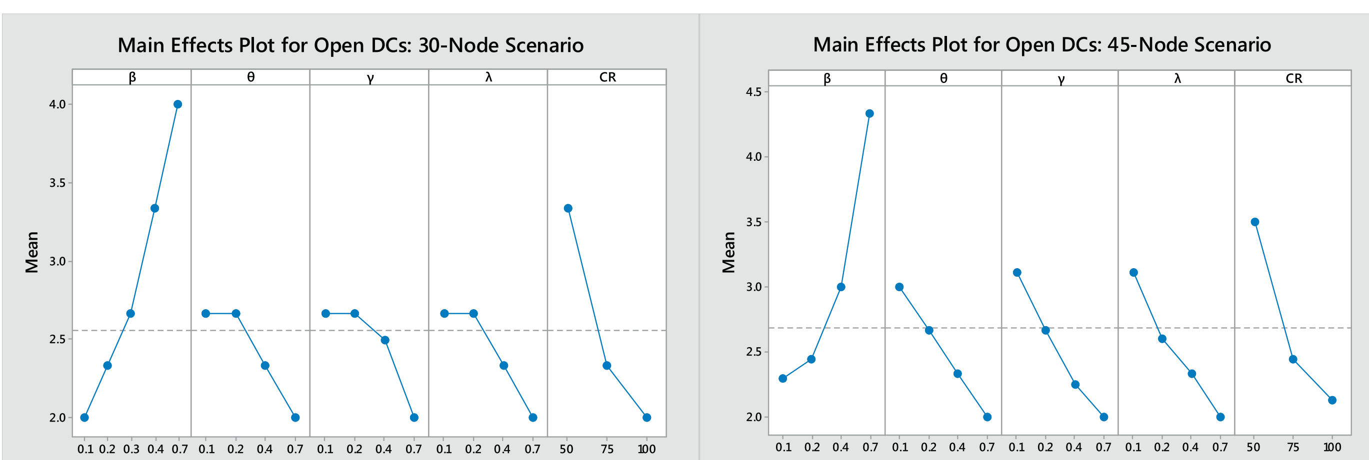

Fig. 3. Effects of model parameter and variable setting on the number of open DCs.

number of open DCs drops from a maximum of 4 to a minimum of 1 when the CR is increased from 50 to 100 miles (Tables 4 and 5).

In Fig. 3, as β (the transportation cost weight) is increased from 0.1 to 0.7, the number of open DCs increases remarkably. On the other hand, when θ, and γ and λ are increased from 0.1 to 0.7, the average number of open DCs decreases, since these weights control the influence of the inventory, the facility location and the lost sales costs, respectively. An analysis of variance was carried out to evaluate the effect model parameters β, θ and γ had on the number of open DCs. For both 30 and 45-node problems, we found that only β significantly affected the number of open DCs (at 95% significance). In fact, when transportation cost has a higher priority (larger β), the system responds by increasing the number of DCs in an effort to minimize the distance traveled within the network. Conversely, when inventory cost has a higher priority (larger

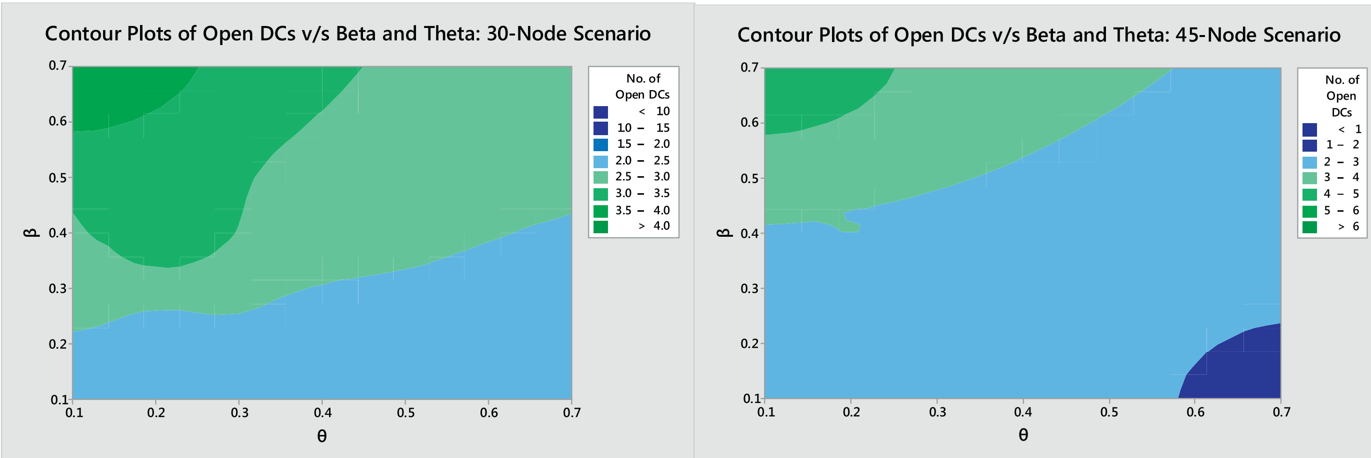

Fig. 4. Contour plots beta and theta v/s system cost and the number of open DCs.

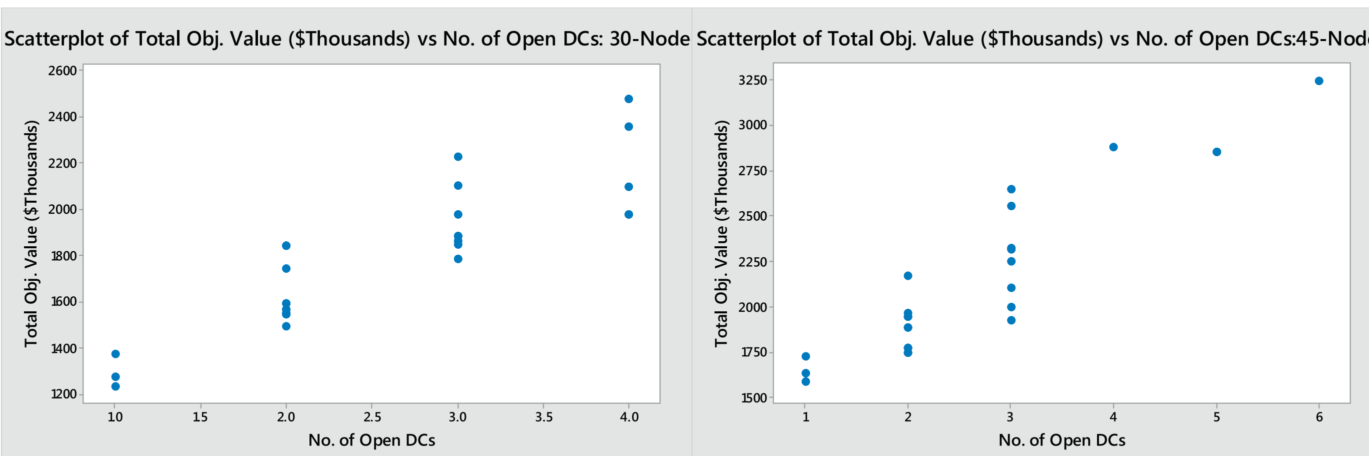

Fig. 5. Scatter plot of total objective value v/s the number of open DCs.

θ), the system reacts by decreasing the number of opened DCs in the network and increasing the reliance upon risk-pooling in the system. Contour plots in Fig. 4 illustrate that parameter β has a relatively higher impact on the number of DCs open and hence the total system cost than does parameter θ. Plots with similar contour plots were realized when β against the other parameters.

It can also be seen from results in Table 4 (30-Node solution) that only jointly increasing the values of θ and γ to 0.7 results in opening a single DC (location 23 when CR is 75 and 100 miles). Additionally, DCs 27 and 53 are consistently open for CR 75 and 100. As expected, the 50 miles CR considerations resulted in the largest number of open DCs (3 to 4), and nearly the same DCs were consistently selected in the 45-node problems (Table 5). Our results also show that the system's cost depends linearly on the number of open DCs as shown in Fig. 5. A result influenced most by the cost tradeoffs in transportation and inventory. That is, opening additional DCs may increase the facility location cost, but it reduces both the inventory cost due to the risk-pooling effects and the overall transportation cost incurred by the retail stores assigned to a particular DC. In operational terms, a UMR is likely to open more DCs for a given range of total cost.

6.2. Tradeoffs in total cost

Given the positive correlation between the number of open DCs and the total system cost, the trend of the total cost with the model parameters was similar to the trend between the number of open DCs with the same parameters (Fig. 3). As expected, an increase in the transportation weight β, dramatically increases the total objective function value. Similarly, only β was found to be significantly influential to the total objective value at 95% confidence. The rest of the parameters slightly reduced the total objective value. A further analysis indicated that there was almost no significant interaction between the model parameters apart from slight interactions between the levels of β at the lowest coverage radius (50-mile CR). This slight interaction, which could be due to a clustering effect of DCs, is expected since a low CR dictates the need for more DCs in the system, and as a result, necessitates more short trips between the retailers and the assigned DCs. Therefore it can be concluded that at lower CR values, the transportation weight ceases to have grater effects on the model solutions. We note that overall, all the experiment runs require less than 600 seconds of CPU time and this provide feasibility for the potential need of real time solutions for tactical decisions.

6.3. The economics of transportation and inventory cost

With reference to our experimental design in Table 2, the reader should recall our use of both θ and β in Eq. (1). They can affect the optimal cost and configuration of the resulting design. This effect is explored in terms of $\sqrt{\theta(1+\beta)}$ (Shen, 2005, Shen & Qi, 2007) and reported below in Table 6 as:

We notice that $\sqrt{\theta(1+\beta)}$ reaches a maximum value of 0.88 when θ is 0.7 and β has its lowest value of 0.1. In all twelve

Table 6
Trends in centralization vs. decentralization.

Set#	β	θ	$\sqrt{\theta(1+\beta)}$	Set#	β	θ	$\sqrt{\theta(1+\beta)}$
#1	0.1	0.1	**0.33**	**#7**	0.2	0.4	**0.69**
#2	0.1	0.7	**0.88**	**#8**	0.2	0.2	**0.49**
#3	0.7	0.1	**0.41**	**#9**	0.4	0.3	**0.65**
#4	0.1	0.1	**0.33**	**#10**	0.1	0.2	**0.47**
#5	0.2	0.2	**0.49**	**#11**	0.2	0.4	**0.69**
#6	0.4	0.2	**0.53**	**#12**	0.3	0.1	**0.36**

scenarios, the value of $\sqrt{\theta(1+\beta)}$ is higher than β or θ individually. Within each setting of coverage radius level, it is observed that decreasing values of $\sqrt{\theta(1+\beta)}$ trend toward opening more DCs (more decentralization). However, when β is much higher than θ, we expect more open DCs in the system as seen in experiments 3, 15, and 27 of Tables 4 and 5. For some mid-values ($\sim$0.4) of β, we saw several cases of slightly less centralization (more open DCs) as the coverage radius decreased. Sets# 7 and 8 also indicate this phenomenon. Set# 9 has a higher β compared to set# 8. But because of $\sqrt{\theta(1+\beta)}$ term (0.65 vs. 0.49, respectively), the model increased the number of DCs when the coverage radius was 50 miles. This highlights the inherent economics of distance, and the interplay between transportation cost and working inventory cost. They inherently influence distribution design decisions made by the model.

7. Research limitations

Location modeling problems described in this paper are common in many traditional retail and manufacturing sectors. However, this paper formulated a novel problem in the non-traditional UMR sector as a three-echelon multi-product distribution system. The particular UMR problem includes the need to decide flows by product type and includes unique types of retail operations, some of which have only outbound product flows while others have both inbound and outbound flows. In practice, the flows are subject to reassignment during a typical year due to seasonal uncertainty so fast solution approaches are of interest. We first formulated the problem as an integer nonlinear programming (INLP) problem. Safety-stock considerations imposed a non-linear objective function rendering the problem NP-hard. Next, a two-step reformulation of the original problem increased attractiveness of deploying a branch-cut and price algorithm. We found the optimal number and locations for facilitating DCs, and the optimal assignment of retail stores and attended donation centers for a set of initial problems. The contributions of the research include the problem definition, its formulation and optimal solution algorithm, and the experimental analysis under relevant problem scenarios facing a large UMR chain.

Our formulation and solution procedure parallels some of the earlier work on LMRP. The computational experiments included 30 and 45-node problem sets with varying settings on the problem parameters i.e., β, θ, γ, and λ, with varying coverage radius (CR) settings. Our results confirm the intuitive expectation that more DCs are open at lower CR levels. Also, our results indicate that increase in β values significantly increased the number of open DCs and the total objective cost value. As expected, placing increased importance (larger β) on the transportation cost led to more decentralization of the distribution system (opened more DCs) in an effort to minimize the total distance traveled. Conversely, an increase in the relative value of θ, increased the importance of inventory cost and therefore led to more centralization (fewer opened DCs) of the distribution system in order to minimize the total inventory cost of the system. This results in higher levels of risk pooling. The results also show that overall; β has a larger effect on the model response variables. In particular, whenever the weight of β is more than or equal 40% relative to the other parameters, the number of open DCs doubled. Our results also indicate that there is a linear relationship between the number of open DCs and the total objective cost value (for the range of open DCs in the study). The successful application of the model and procedure on the UMR problem supports its managerial relevance. Several practical scenarios were tested under real-world contexts using data resembling the test data used here. Initial analysis demonstrated potential annual savings of just over 700,000 dollars vs. current operating practice.

As with any research study, there are limitations. First, while it is possible to evaluate larger-sized data sets, this would likely require additional computational resources and consideration of other algorithmic approaches. It is also possible to experiment with additional settings on the experimental factors. Imposing capacity restrictions to the candidate DCs and allowing vehicle routing options for transportation are two additional extensions of the model presented in this paper. Another natural extension to the proposed model would be to formulate the model as a dynamic programming problem. This extension is important because it could render the model robust enough to incorporate product seasonality into the network. For instance, the average donations and demands for each product-type may easily vary both by season and geographically. In addition, considering tactical and operational decision variables may be allowed to change within a specific time horizon. These variables include: retailer assignments, working inventory level in DCs, safety stock level in DCs, and vehicle routing. Each of these represents possible future extensions of the UMR problem introduced in this paper.

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Supplementary materials

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